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Research Article

Couple Stress

Flow Analysis of a Couple Stress Fluid Through Porous Media in the Absence of a Pressure Gradient

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This study investigates the steady flow of a chemically reacting couple-stress fluid through a porous medium without an imposed pressure gradient. Using the second law of thermodynamics, we analyze entropy generation and thermal irreversibility in the system. The higher-order differential equations that govern the flow, incorporating couple stresses and porous permeability effects, are non-dimensionalized and simplified to obtain approximate analytical solutions. Key parameters such as the stress parameter of the couple and the permeability of the porous medium are examined to determine their influence on flow behavior and the rates of entropy generation. The results provide insights relevant to the optimization of heat and mass transfer in complex fluid systems with applications in chemical and thermal engineering.

Keywords: fluid, pressure, entropy, thermodynamics

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1. Introduction

The second law of thermodynamics plays a crucial role in design and optimization of thermal systems by quantifying entropy generation, which reflects irreversibility and energy losses. Minimizing entropy generation improves system efficiency and exergy, as established by Bejan (1982) and subsequent research [3, 4]. Entropy generation analysis, based on second law, offers a more accurate evaluation of system performance than first-law methods, especially in complex fluid flows [1, 5]. Non-Newtonian fluids, including suspensions, polymer solutions, and biological fluids, exhibit behaviors that deviate from Newtonian models, necessitating advanced constitutive models. Couple stress fluid model, introduced by Stokes, accounts for microstructural effects such as couple stresses and asymmetric stress tensors, making it suitable for fluids with size-dependent properties [7].

Previous studies have explored generation of entropy in couple stress fluids flowing through porous media under various conditions [8, 9]. However, limited attention has been given to combined effects of chemical reactions and cross-diffusion phenomena (e.g., Dufour effect) on entropy generation in such flows. This work addresses that gap by analyzing thermodynamic irreversibility in chemically reacting couple stress fluids flowing steadily through porous channels.

2. Statement of the Problem

The governing equation guiding the thermodynamic analysis of couple stress fluid flow through a porous medium is a fourth-order differential equation coupled with boundary conditions:

$$0 = \mu \frac{d^2\bar{u}}{d\bar{y}^2} - \eta \frac{d^4\bar{u}}{d\bar{y}^4} - \frac{\mu\bar{u}}{K}. \quad (2.1)$$

The corresponding boundary conditions are given by:

$$\bar{u} = \frac{d^2\bar{u}}{d\bar{y}^2} = 0 \quad at\ \bar{y} = 0, \qquad \bar{u} = \frac{d^2\bar{u}}{d\bar{y}^2} = 1 \quad at\ \bar{y} = h.$$

We introduce the following dimensionless variables and parameters:

$$y = \frac{\bar{y}}{h}, \qquad u = \frac{\bar{u}}{v_0}, \quad \lambda^2 = \frac{\mu h^2}{\eta}, \qquad v = \frac{\mu}{\rho},$$

where h is the channel half-width, u and $\bar{u}$ are dimensionless and dimensional axial velocities, respectively, μ is the dynamic viscosity, η is the couple stress viscosity, ρ is the fluid density, and K is the permeability of porous medium [11, 12, 13].

The study of couple stress fluid flow in porous media has been extensively discussed by Adesanya and Makinde [11, 14], who analyzed entropy generation and convective heating effects in such flows.

The modeling approach aligns with the formulations provided by Anderson [15, 2] and Hayat et al. [17], where higher-order derivatives account for viscoelastic effects. Similar boundary conditions have been applied in related works on flow through porous channels [12].

3. Nondimensionalization and Reformulation

To nondimensionalize the governing equation, we substitute:

$$\bar{u} = u v_0, \qquad y = \frac{\bar{y}}{h}, \qquad \frac{d}{d\bar{y}} = \frac{1}{h}\frac{d}{dy}.$$

Hence, the derivatives transform as follows:

$$\frac{d\bar{u}}{d\bar{y}} = \frac{v_0}{h}\frac{du}{dy}, \qquad (3.1)$$

$$\frac{d^2\bar{u}}{d\bar{y}^2} = \frac{v_0}{h^2}\frac{d^2u}{dy^2}, \qquad (3.2)$$

$$\frac{d^4\bar{u}}{d\bar{y}^4} = \frac{v_0}{h^4}\frac{d^4u}{dy^4}, \qquad (3.3)$$

Substituting $(3.1) - (3.3)$ into (2.1) we obtain:

$$0 = \frac{\mu v_0}{h^2}\frac{d^2u}{dy^2} - \frac{\eta v_0}{h^4}\frac{d^4u}{dy^4} - \frac{\mu v_0 u}{K} \qquad (3.4)$$

Dividing through by $\frac{\mu v_0}{h^2}$, we get:

$$0 = \frac{d^2u}{dy^2} - \frac{\eta}{\mu h^2}\frac{d^4u}{dy^4} - \frac{h^2 u}{K}. \qquad (3.5)$$

Let,

$$a = \frac{1}{\lambda^2}, \ \ \beta = \frac{h^2}{K} \qquad (3.6)$$

(3.5) becomes:

$$\frac{d^2u}{dy^2} - a\frac{d^4u}{dy^4} - \beta u = 0. \qquad (3.7)$$

Rewriting in a standard form, we obtain the nondimensional momentum equation:

$$a\frac{d^4u}{dy^4} - \frac{d^2u}{dy^2} + \beta u = 0, \qquad (3.8)$$

where a is the couple stress parameter and β is the porous permeability constant.

4. Solution of the Problem

The differential equation is

$$a\frac{d^4u}{dy^4} - \frac{d^2u}{dy^2} + \beta u = 0$$

Assume a solution of the form

$$u(y) = e^{my} \qquad (4.1)$$

Differentiating (4.1) we obtain:

$$\frac{du}{dy} = u'(y) = me^{my}. \qquad (4.2)$$

Then, the second, third, and fourth derivatives of *u(y)* are given by:

$$\frac{d^2u}{dy^2} = u''(y) = m^2e^{my}. \qquad (4.3)$$

$$\frac{d^3u}{dy^3} = u'''(y) = m^3e^{my}. \qquad (4.4)$$

$$\frac{d^4u}{dy^4} = u^{(iv)}(y) = m^4e^{my}. \qquad (4.5)$$

Substitute into (3.8) :

$$am^4e^{my} - m^2e^{my} + \beta e^{my} = 0$$

$$\Longrightarrow e^{my}(am^4 - m^2 + \beta) = 0. \qquad (4.6)$$

Since $e^{my} \neq 0$, we have

$$am^4 - m^2 + \beta = 0. \qquad (4.7)$$

Let $k = m^2$ Then (4.7) becomes

$$ak^4 - k^2 + \beta = 0,$$

Using the quadratic formula:

$$k = \frac{-(-1)\pm\sqrt{(-1)^2-4a\beta}}{2a} = \frac{1\pm\sqrt{1-4a\beta}}{2a}$$

Recall that $k = m^2$, so

$$m = \pm\sqrt{k} = \pm\sqrt{\frac{1\pm\sqrt{1-4a\beta}}{2a}}.$$

Therefore, the general solution is

$$u(y) = Ae^{m_1y} + Be^{m_2y} + Ce^{m_3y} + De^{m_4y}$$

where

$$m_1 = \sqrt{\frac{1-\sqrt{1-4a\beta}}{2a}},$$

$$m_2 = -\sqrt{\frac{1-\sqrt{1-4a\beta}}{2a}},$$

$$m_3 = \sqrt{\frac{1+\sqrt{1-4a\beta}}{2a}},$$

$$m_4 = -\sqrt{\frac{1+\sqrt{1-4a\beta}}{2a}}.$$

Using the boundary conditions:

At $y = 0, u(0) = 0;$

$$A + B + C + D = 0.$$

At $y = 1, u(1) = 1;$

$$Ae^{m_1} + Be^{m_2} + Ce^{m_3} + De^{m_4} = 1.$$

At $y = 0, u''(0) = 0;$

$$Am_1^2 + Bm_2^2 + Cm_3^2 + Dm_4^2 = 0$$

At $y = 1, u''(1) = 1;$

$$Am_1^2e^{m_1} + Bm_2^2e^{m_2} + Cm_3^2e^{m_3} + Dm_4^2e^{m_4} = 1$$

Bringing the boundary conditions together, we solve the system:

$$\begin{aligned} A + B + C + D &= 0, \\ Ae^{m_1} + Be^{m_2} + Ce^{m_3} + De^{m_4} &= 1, \\ Am_1^2 + Bm_2^2 + Cm_3^2 + Dm_4^2 &= 0, \\ Am_1^2e^{m_1} + Bm_2^2e^{m_2} + Cm_3^2e^{m_3} + Dm_4^2e^{m_4} &= 1. \end{aligned}$$

Solving the system, the constants A, B, C, D are given by,

$$A = \frac{e^{m_1+m_3}\left(-2a(-e^{m_1} + e^{m_1+2m_3}) - 2e^{m_1} + e^{m_1+2m_3}\left(1 + \sqrt{1-4a\beta}\right)\right)}{\sqrt{1-4a\beta}(e^{m_1+m_3} - 2e^{m_1+3m_3} - 2e^{3m_1+m_3} + 2e^{3m_1+3m_3})},$$

$$B = -\frac{e^{m_1}\left(1 - 2a + \sqrt{1-4a\beta}\right)}{2(e^{2m_1} - 1)\sqrt{1-4a\beta}},$$

$$C = \frac{e^{m_3}\left(1 - 2a + \sqrt{1-4a\beta}\right)}{2(e^{2m_3} - 1)\sqrt{1-4a\beta}},$$

$$D = -\frac{e^{m_3}\left(1 - 2a + \sqrt{1-4a\beta}\right)}{2(e^{2m_3} - 1)\sqrt{1-4a\beta}},$$

Therefore,

$$\begin{aligned} u(y) = &\frac{e^{m_1+m_3}\left(-2a(-e^{m_1} + e^{m_1+2m_3}) - 2e^{m_1} + e^{m_1+2m_3}\left(1 + \sqrt{1-4a\beta}\right)\right)}{\sqrt{1-4a\beta}(e^{m_1+m_3} - 2e^{m_1+3m_3} - 2e^{3m_1+m_3} + 2e^{3m_1+3m_3})} e^{m_1 y} \\ &- \frac{e^{m_1}\left(1 - 2a + \sqrt{1-4a\beta}\right)}{2(e^{2m_1} - 1)\sqrt{1-4a\beta}} e^{m_2 y} \\ &+ \frac{e^{m_3}\left(1 - 2a + \sqrt{1-4a\beta}\right)}{2(e^{2m_3} - 1)\sqrt{1-4a\beta}} e^{m_3 y} \\ &- \frac{e^{m_3}\left(1-2a+\sqrt{1-4a\beta}\right)}{2(e^{2m_3}-1)\sqrt{1-4a\beta}} e^{m_4 y}. \quad (4.8) \end{aligned}$$

5. Results and Analysis

This section presents numerical results for the velocity distribution u(y) of a steady, chemically reacting couple-stress fluid flowing through a porous channel. The analysis focuses on the effects of the couple stress parameter *a* and the porous permeability parameter β. The boundary conditions $u(0) = 0 \; and \; u(1) = 1$ are enforced throughout.

5.1. Effect of Couple Stress Parameter

Table 1 and Figure 1 illustrate the variation of the velocity profile *u(y)* for different values of the couple stress parameter *a*, at a fixed porous permeability parameter $\beta = 0.3.$

y	a = 0.01	a = 0.05	a = 0.5
0.0	0.00000	0.00000	0.00000
0.1	9.45344 x 10 - 2	9.21226 x 10 - 2	8.58026 x 10 - 2
0.2	1.89354 x 10 - 1	1.84603 x 10 - 1	1.72411 x 10 - 1
0.3	2.84746 x 10 - 1	2.77814 x 10 - 1	2.60642 x 10 - 1
0.4	3.81003 x 10 - 1	3.72166 x 10 - 1	3.51363 x 10 - 1
0.5	4.78425 x 10 - 1	4.68131 x 10 - 1	4.45363 x 10 - 1
0.6	5.77338 x 10 - 1	5.66282 x 10 - 1	5.43644 x 10 - 1
0.7	6.78129 x 10 - 1	6.67352 x 10 - 1	6.47154 x 10 - 1
0.8	7.81341 x 10 - 1	7.72318 x 10 - 1	7.56943 x 10 - 1
0.9	8.87936 x 10 - 1	8.82547 x 10 - 1	8.74145 x 10 - 1
1.0	1.00000	1.00000	1.00000

Table 1: Velocity profile *u(y)* for various values of couple stress parameter *a* with $\beta = 0.3.$

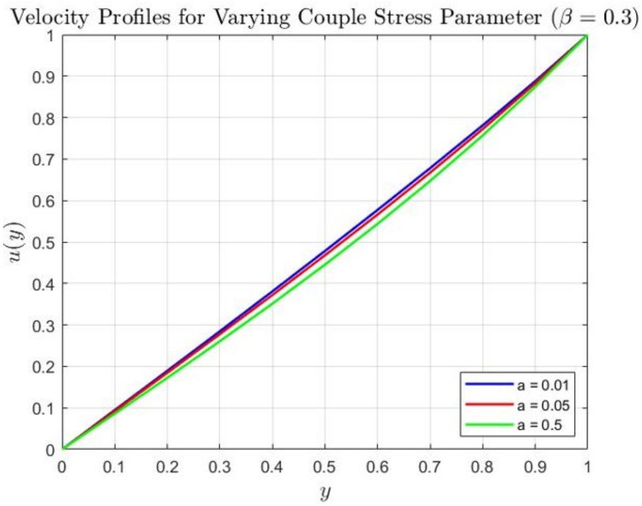

Figure 1: Velocity distribution *u(y)* for varying couple stress parameter *a at* $\beta = 0.3.$ Increasing *a* reduces peak velocity near channel centerline due to enhanced internal microstructural resistance.

The results indicate that as the couple stress parameter increases, the velocity decreases throughout the domain, particularly near the centerline where the flow is most active. This is attributed to the augmented viscous resistance induced by couple stresses, which inhibit shear deformation due to micro-rotational effects.

5.2. Effect of Porous Permeability Parameter

Table 2 and Figure 2 show the influence of the porous permeability parameter *β* on the velocity distribution for a fixed couple stress parameter *β at a* = 0.01.

y	β = 1	β = 2	β = 3
0.0	0.00000	0.00000	0.00000
0.1	8.5108 × 10−2	7.36722 × 10−2	6.4144 × 10−2
0.2	1.71076 × 10−1	1.4885 × 10−1	1.30276 × 10−1
0.3	2.58775 × 10−1	2.27068 × 10−1	2.00444 × 10−1
0.4	3.49089 × 10−1	3.09918 × 10−1	2.76809 × 10−1
0.5	4.42933 × 10−1	3.99077 × 10−1	3.61705 × 10−1
0.6	5.41254 × 10−1	4.96318 × 10−1	4.57661 × 10−1
0.7	6.45045 × 10−1	6.03493 × 10−1	5.67378 × 10−1
0.8	7.55353 × 10−1	7.2243 × 10−1	6.93519 × 10−1
0.9	8.73282 × 10−1	8.54579 × 10−1	8.38007 × 10−1
1.0	1.00000	1.00000	1.00000

Table 2: Velocity profile *u(y)* for different values of permeability parameter *β at a = 0.01*.

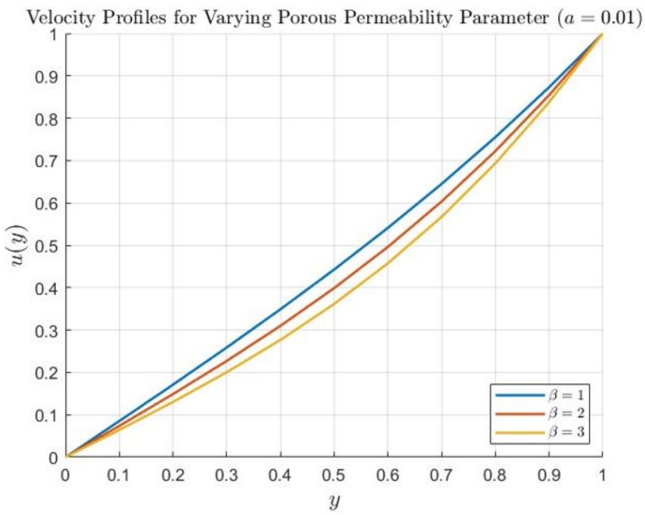

Figure 2: Variations in porous permeability parameter β significantly affect velocity profile *u(y)* when couple stress parameter is fixed at *a = 0.01* Specifically, as β increases, velocity decreases due to increased flow resistance induced by porous medium. It is observed that increasing β results in a suppressed velocity profile across domain.

This behavior reflects enhanced resistance imposed by porous structure, which retards fluid motion & reduces kinetic energy, consistent with physical expectations.

5.3. Interpretation and Physical Implications

Interplay between couple stress and porous permeability effects significantly alters the flow dynamics. Higher values of intensify internal rotational resistance, increasing viscous dissipation & thus reducing fluid velocity. Similarly, increased β reflects a denser porous medium, leading to stronger drag forces & diminished flow rates. These mechanisms are critical in applications involving suspensions, polymeric fluids, and industrial slurries, where both microstructure & porous environments govern performance. Observed trends conform with thermodynamic principles, particularly in terms of entropy generation & energy dissipation, highlighting non-negligible influence of both parameters on system irreversibility [16, 10, 6].

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